SIMULTANEOUS DETERMINATION OF TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY AND VOLUMETRIC HEAT CAPACITY

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A method is presented for determining the thermal conductivity and the volumetric heat capacity when they depend on the temperature. A block diagram of the program is given with the results of determining the thermophysical characteristics of heat-insulating materials.

A method is presented for simultaneously determining the thermal conductivity and the volumetric heat capacity of materials when they vary linearly with the temperature:

$$\lambda = A + Bt, \; c\gamma = K + Dt.$$

The essence of the method consists in comparing the experimental and calculated temperatures in the sample. For given values of the thermal conductivity and the volumetric heat capacity the nonstationary heat-conduction equation is solved with the boundary conditions which were maintained in the experiment. A comparison of the experimental and calculated temperature curves shows a discrepancy, and the values of the thermal conductivity and the volumetric heat capacity are chosen to minimize the maximum discrepancy. Then the $\lambda(t)$ and $c\gamma(t)$ which correspond to the calculated temperatures deviating least from the experimental values will best approximate the thermal conductivity and the volumetric heat capacity of the sample under study.

The problem reduces to finding the minimum of $F(\lambda, c\gamma)$, where

$$F(\lambda, \; c\gamma) = \max |t^{c}_{j,\xi} - t^{e}_{j,\xi}|. \tag{1}$$

The experiment was performed by the Krisher–Esdorn scheme, with eight samples to ensure symmetric heating of the specimen under study [1]. The experiment was arranged so that on one surface (X 0) of the sample $q_1 = \text{const}$, and on the other ($X = l$) $q_2 = 0$. During the heating the readings of the thermocouples at the surfaces $X = 0$ and $X = l$, i.e., the temperatures $t^{e}_{0,\tau} = f_1(\tau)$ and $t^{e}_{l,\tau} = f_2(\tau)$, were recorded. The experiment was stopped when the temperature on the heated surface increased to two or three times its initial value. Thus from the experimental data we have for the surface $X = 0$ the values $t^{e}_{0,\tau} = f_1(\tau)$, $q_1 = \text{const}$, and for the surface $X = l$ the values $t^{e}_{l,\tau} = f_2(\tau)$, $q_2 = 0$; i.e., $t^{e}_{j,\xi}$ will correspond to the values of the temperatures $t^{e}_{0,\tau}$, $t^{e}_{l,\tau}$ on the two boundary surfaces $X = 0$ and $X = l$ in a given time interval.

The values of $t^{c}_{0,\tau}$ and $t^{c}_{l,\tau}$ corresponding to $t^{c}_{j\xi}$ are calculated from the equation

$$c\gamma(t)\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left[\lambda(t)\frac{\partial t}{\partial x}\right] \tag{2}$$

and the boundary conditions

$$\lambda(t)\frac{\partial t}{\partial x}\bigg|_{x=0} = q_1; \quad \frac{\partial t}{\partial x}\bigg|_{x=l} = 0. \tag{3}$$

Equation (2) and boundary conditions (3) are written in finite-difference form and solved for the temperature

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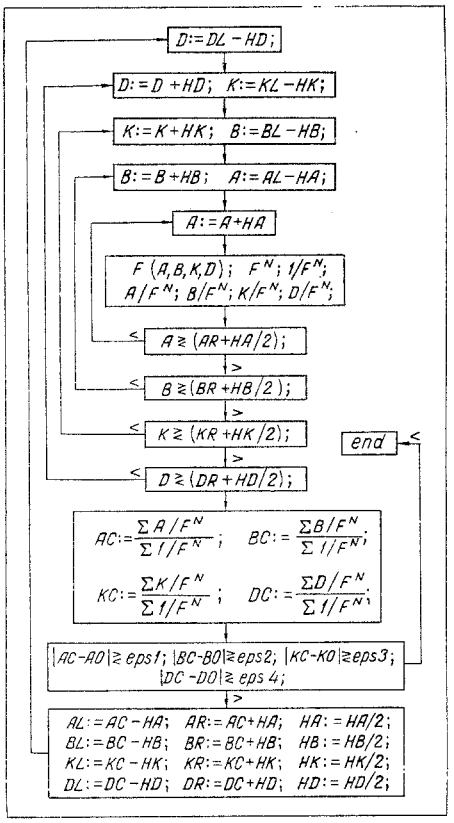

Fig. 1. Block diagram of program for finding the thermal conductivity $\lambda = A + Bt$ and the volumetric heat capacity $c\gamma = K + Dt$.

by the pivot method after specifying $\lambda(t)$ and $c\gamma(t)$.

After the experimental and calculated temperatures are found an optimization process is begun. The minimax (1) is found by varying the required quantities A, B, K, and D determining the thermal conductivity and the volumetric heat capacity. To do this the ranges of the quantities A_{min}, A_{max}, B_{min}, B_{max}, K_{min}, K_{max}, D_{min}, D_{max} are specified and then each of them is varied from its minimum to maximum values in specified steps with the other quantities held constant. Of course $F(\lambda, c\gamma)$ is found for each new value of one of the coefficients. From all the values obtained for $F(\lambda, c\gamma)$ the minimum is chosen. This value will also be optimum and the quantities A, B, K, and D corresponding to it are optimum in the expressions for the required $\lambda = A + Bt$ and $c\gamma = K + Dt$. From these values new narrower ranges of variation of the quantities A, B, K, and D are chosen etc. By successively decreasing the ranges of variation of A, B, K, and D the values of $\lambda(t)$ and $c\gamma(t)$ are obtained with the prescribed accuracy.

The program for finding the minimax (1) is written in ALGOL; the block diagram is shown in Fig. 1.

Before performing experiments on the samples of materials the program was tested in the following way. The temperatures obtained by solving Eq. (2) with boundary conditions (3) and $\lambda = 0.058 + 0.0058t$ W/m·deg and $c\gamma = (83.7 + 0.418t) \times 10^3$ J/m³·deg in the range 15.5-126°C were taken as the experimental temperature distribution. The minimax (1) was found for the following ranges of variation of the quantities: for A, 0.035-0.093, for B, 0.0035-0.0093, for K, 41.8×10^2-$167.5 \cdot 10^3$, for D, $0.41 \cdot 10^3$-$1.67 \cdot 10^3$. A 37-min optimization process gave optimum values for the thermal conductivity of $A_{opt} = 0.058$, B_{opt} 0.0058, and for the volumetric heat capacity $K_{opt} = 83.7 \cdot 10^3$, $D_{opt} = 0.418 \cdot 10^3$. That is, there is complete agreement between the unknown and the specified values. This occurred for a wide range of the specified initial values of A, B, K, and D.

Experiments were performed on 250 × 250 mm samples of cellular polyurethane (PPU-3S) 50 mm thick for symmetric heating with $q_1 = 36.5$ W/m² in the temperature range 19-76°C, on samples of phosphate styrene cellular plastic (PSP) 40 mm thick for heating with $q_1 = 34.9$ W/m² in the temperature range of 20 to 15°C. and on samples of mineralized slabs of increased rigidity 40 mm thick for heating with q = 34.9 W/m² in the 18-62°C temperature range.

For PPU-3S the optimization process gave a thermal conductivity $\lambda = 0.036 + 0.00023t$ W/m·deg and a volumetric heat capacity $c\gamma = (91.69 + 1.256t) \cdot 10^3$ J/m³·deg for PSP-233 $\lambda = 0.0485 + 0.000238t$ W/m·deg and $c\gamma = (155.16 + 1.256t) \cdot 10^3$ J/m³·deg; for mineralized slabs $\lambda = 0.043 + 0.00024t$ W/m·deg and $c\gamma = (160.77 + 1.465t \cdot 10^3$ J/m³·deg.

For comparison the samples were tested by the Krisher-Esdorn method for temperature changes small enough so that changes in thermophysical properties could be neglected. For example for PPU-3S at $t_{av} = -10$°C, $\lambda = 0.0342$ W/m·deg and $c\gamma = 114.71 \cdot 10^3$ J/m³ deg; at $t_{av} = -27$°C, $\lambda = 0.0314$ W/m·deg and $c\gamma = 104.67 \cdot 10^3$ J/m³·deg.

On the basis of the results obtained we can recommend the method described for finding the temperature dependence of the thermal conductivity and the volumetric heat capacity. In one experiment requiring from 0.5 to 2 h the method yields simultaneously the values of the thermophysical characteristics as a function of temperature.

The simple computational procedure makes it possible to assume initial approximations of the unknown quantities over a wide range. Thus the method has advantages over earlier procedures.

NOTATION

$c\gamma$	is the volumetric heat capacity;
λ	is the thermal conductivity;
q	is the heat flux density;
t	is the temperature;
l	is the thickness of sample;
τ	is the time;
A, B, K, D	are the constants determining the volumetric heat capacity and the thermal conductivity.

LITERATURE CITED

1. A. V. Lykov (editor), Methods of Determining Thermal Conductivity and Thermal Diffusivity [in Russian], Énergiya, Moscow (1973).